ON A NEUTRON POINT KINETIC MODEL CONSIDERING THE VARIATION IN THE FUEL COMPOSITION WITH CONVERGENCE ANALYSIS

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Summary

- Introduction
- Objectives
- Methodology
- Results
- Conclusion

Introduction

Classic Point kinetic model (short time scale).

Reactor fuel stays from 1 up to 3 years in the reactor core (long time scale).

Changes in the nuclear fuel material and properties over time.

Depletion codes - Quasi-static method : linearization of the problem, neutron flux approximation to constant in each time step.

Introduction

New model: Point kinetics and isotope depletion with reactivity decomposition in short and long time scales. Solved the nonlinear equation by the Adomian decomposition method. Considered only neutron poisons in the analysis.

Solves short, intermediate and long time scales together by the same method.

Does not need to approximate the neutron flux to a constant.

Objectives

Expand the model proposes to account Uranium-235 burn-up and transuranium (Plutonium-239 and Neptunium-239).

Convergence analysis of the solution.

Methodology

Classic point kinetic model:

$$\frac{d}{dt}n(t) = \frac{\rho(t) - \bar{\beta}}{\Lambda}n(t) + \lambda C(t),$$

$$\frac{d}{dt}C(t) = \frac{\bar{\beta}}{\Lambda}n(t) - \lambda C(t).$$

Methodology

Isotope concentration equation:

$$\frac{d}{dt}G_i(t) = \underbrace{\gamma_i v \Sigma_f n(t) + \lambda_j G_j(t)}_{\text{increase concentration}} - \underbrace{v\sigma_{x,i} G_i(t) n(t) - \lambda_i G_i(t)}_{\text{decrease concentration}}$$

Methodology

Decomposition of the reactivity:

$$\rho(t) = \underbrace{\rho_s(t)}_{\text{short time scale}} + \underbrace{\rho_l(t)}_{\text{long time scale}}$$

Methodology

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Changes by the reactor operation maneuvers and control.

Variation of nuclear fuel composition.

Methodology

Simplifications:

- 1 energy group.
- 1 precursors group.
- Plutonium-239 delayed neutrons were not considered.
- Uranium-238 concentration and diffusion coefficient were considered constants.
- Fission cross-section is considered constant in the short time scale.

*These simplifications are not mandatory for the method. The model can be expanded.

Methodology

$$\frac{d}{dt}\mathbf{Y} = \underbrace{\mathbf{AY}}_{\text{Linear}} + \underbrace{\mathbf{NY}}_{\text{Non-linear}}$$

Adomian Decomposition Method:

- **Analytical approximation to the solution by an infinite series.**
- **Solutions are obtained by solving recursively linear systems with source terms.**
- **The nonlinear term in the original equation is accounted as the source term.**
- **Nonlinear contributions are based on previous recursions solutions and calculated by Adomian Polynomials.**

Methodology

The recursion initialization is a homogeneous linear system with the initial conditions.

$$\begin{cases} \frac{d}{dt}\mathbf{Y_0} = \mathbf{A}\mathbf{Y_0} \\ \mathbf{Y_0}(0) = \mathbf{Y_I} \end{cases}$$

Analytical solution:

$$\mathbf{Y_0}(\mathbf{t}) = e^{\mathbf{A}t}\mathbf{Y_I}$$

Methodology

All other recursions:

$$\begin{cases} \frac{d}{dt}\mathbf{Y_j} = \mathbf{A}\mathbf{Y_j} + \mathbf{F_{j-1}} \\ \mathbf{Y_j}(\mathbf{0}) = 0 \end{cases}$$

Analytical solution:

$$\mathbf{Y_j}(\mathbf{t}) = e^{\mathbf{A}t}\cancelto{0}{\mathbf{Y_j}(\mathbf{0})} + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{F_{j-1}}(\tau)d\tau$$

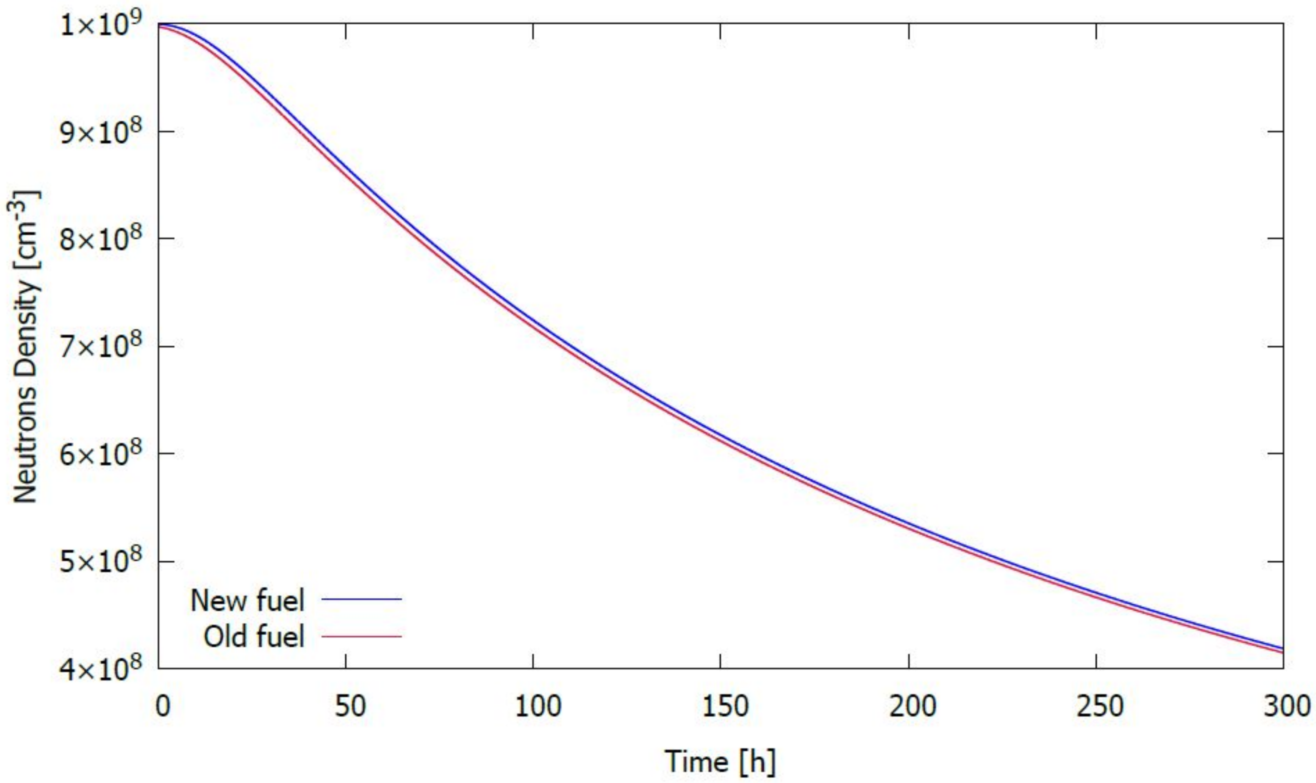
Results

New and old fuel as initial conditions.

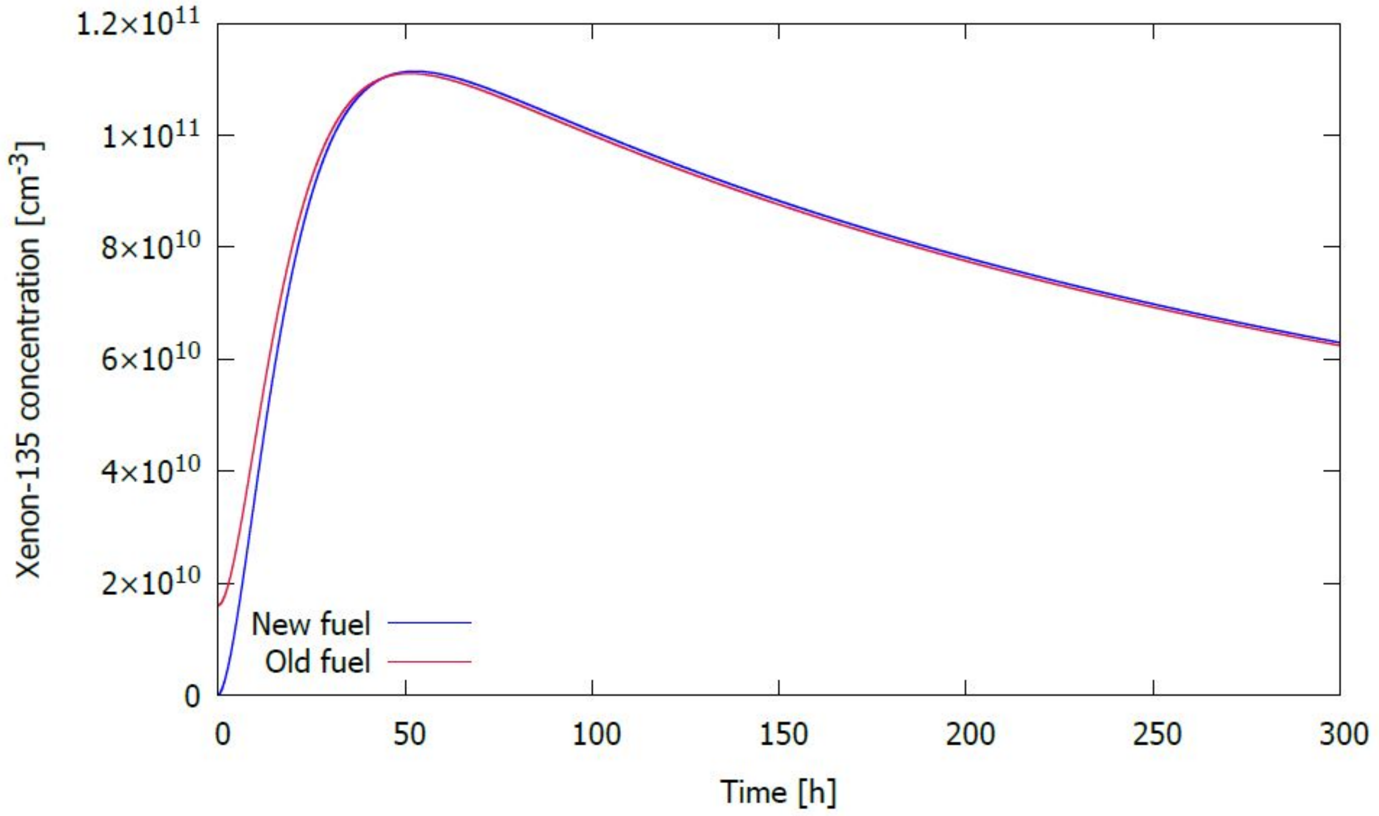
Same neutron density and precursors concentration.

Short time scale reactivity is set critical. $\rho_s(t) = 0$

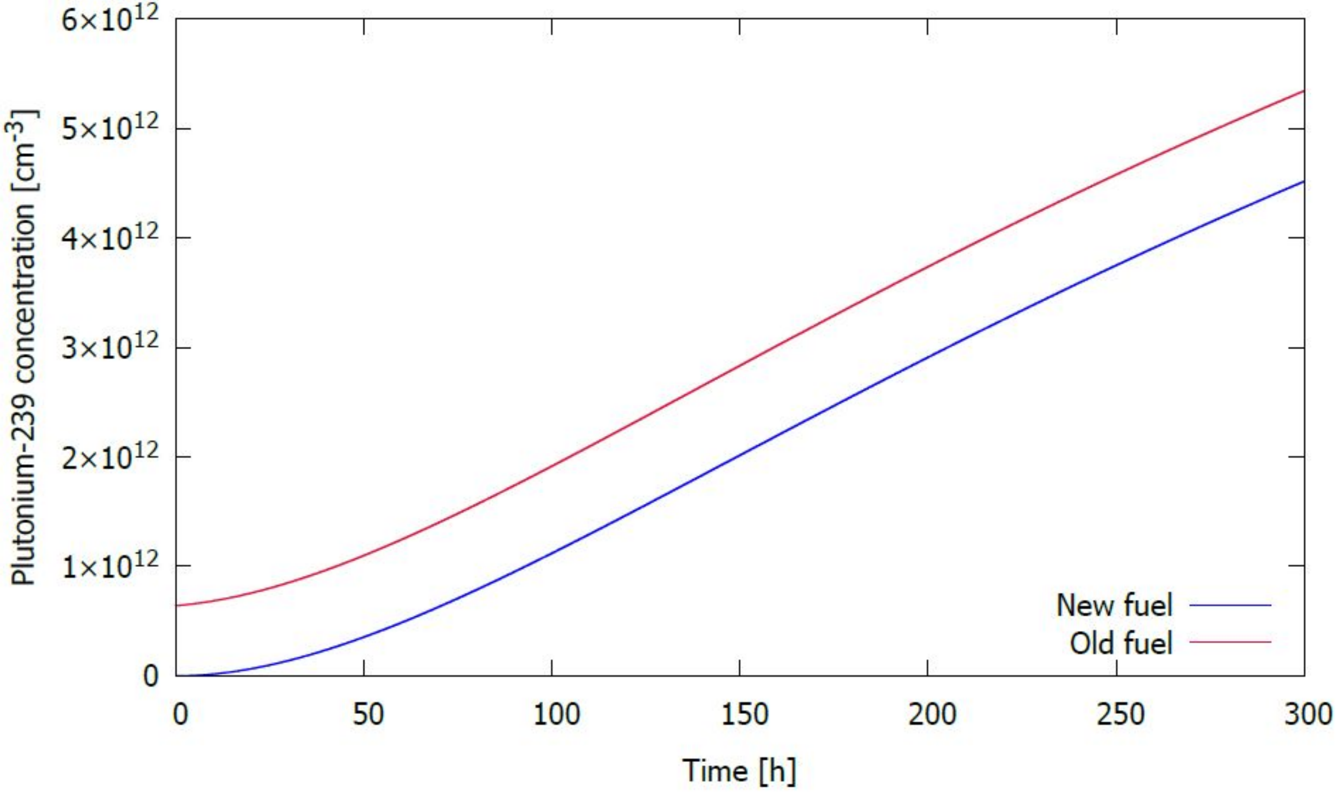
Neutron density



Xenon-135 concentration



Plutonium-239 concentration



Convergence Analysis

The behavior of the solution indicates stability, however, it does not ensure it.

Residual term tends to a constant value orders of magnitude lower than the physical system.

Stiff system - Presents numerical and arithmetical instability. Parameters used were smaller than usual in point kinetic simulations.

Conclusion

This work contributes to expand this new approach to solve the classical problem of point kinetics and isotope composition variation in the nuclear fuel without using simplications in the reactivity and neutron flux, presenting a semi-analytical solution.

The Adomian decomposition method was used to solve the system of nonlinear differential equations. The results are consistent with those reported in the literature and what is expected from a physical point of view.

Conclusion

The solutions obtained through recursions present an apparently convergent behavior, however requires a more extensive analysis in order to better evaluate the convergence of the series.

The stiffness of the system causes computational instability, unabeling the simulation with higher initial condition parameters. Future works currently under development focus to solve this problem.

Conclusion

Future works:

Implement the code in a more suitable language, which has a better floating-point and arithmetic precision (C++).

More precise integration algorithms.

Multi-step Adomian Decomposition Method.

Thank you!

Other results presented in the Progress in Nuclear Energy:
https://doi.org/10.1016/j.pnucene.2019.103134

Other questions: tpaganin@gmail.com

APPENDIX

Methodology

$$\rho(t) \equiv \frac{1}{F} \int_V \int_0^\infty \phi^*(\mathbf{r}, E, t_0) \Bigg(\int_0^\infty [\chi(E')\nu(E')\delta\Sigma_f(\mathbf{r}, E', t)$$
$$+ \delta\Sigma_s(\mathbf{r}, E' \rightarrow E, t)] f(\mathbf{r}, E') dE' + \nabla \cdot [\delta D(\mathbf{r}, E, t) \nabla f(\mathbf{r}, E)]$$
$$- \delta\Sigma_t(\mathbf{r}, E, t) f(\mathbf{r}, E) \Bigg) dE d^3\mathbf{r},$$

$$\delta\Sigma_x(\mathbf{r}, E, t) \equiv \Sigma_x(\mathbf{r}, E, t) - \Sigma_x(\mathbf{r}, E, t_0) = \sum_{i=1}^{I} [G_i(\mathbf{r}, t) - G_i(\mathbf{r}, t_0)] \sigma_x^i(E)$$

Methodology

Fission

$$\rho(t) \equiv \frac{1}{F} \int_V \int_0^\infty \phi^*(\mathbf{r}, E, t_0) \Bigg(\int_0^\infty [\boxed{\chi(E')\nu(E')\delta\Sigma_f(\mathbf{r}, E', t)}$$

Scattering

$$\boxed{+ \delta\Sigma_s(\mathbf{r}, E' \to E, t)}] f(\mathbf{r}, E') dE' + \nabla \cdot [\boxed{\delta D(\mathbf{r}, E, t)} \nabla f(\mathbf{r}, E)]$$

Diffusion

$$\boxed{- \delta\Sigma_t(\mathbf{r}, E, t)} f(\mathbf{r}, E) \Bigg) dE d^3\mathbf{r},$$

Total cross-section

Isotope Concentration

$$\delta\Sigma_x(\mathbf{r}, E, t) \equiv \Sigma_x(\mathbf{r}, E, t) - \Sigma_x(\mathbf{r}, E, t_0) = \sum_{i=1}^{I} [\boxed{G_i(\mathbf{r}, t) - G_i(\mathbf{r}, t_0)}] \sigma_x^i(E)$$

Source term

$$\mathbf{F_j(t)} = \begin{bmatrix} \Big((\nu_{Pu}-1)\sigma_f^{Pu}A_j^{Pu} + (\nu_u-1)\sigma_f^{U235}A_j^{U235} - \sigma_c^{Xe}A_j^{Xe} - \sigma_c^{Sm}A_j^{Sm} \\ -(\nu_u-1)\sigma_f^{U235}B_j^{U235} + \sigma_c^{Xe}B_j^{Xe} + \sigma_c^{Sm}B_j^{Sm} - (\nu_{Pu}-1)\sigma_f^{Pu}B_j^{Pu} \Big) \\ 0 \\ \gamma_I^U\sigma_f^U A_j^{U235} + \gamma_I^{Pu}\sigma_f^{Pu}A_j^{Pu} \\ \gamma_{Xe}^U\sigma_f^U A_j^{U235} + \gamma_{Xe}^{Pu}\sigma_f^{Pu}A_j^{Pu} - \sigma_c^{Xe}A_j^{Xe} \\ \gamma_{Pm}^U\sigma_f^U A_j^{U235} + \gamma_{Pm}^{Pu}\sigma_f^{Pu}A_j^{Pu} \\ -\sigma_c^{Sm}A_j^{Sm} \\ -\sigma_f^{U235}A_j^{U235} \\ -\sigma_c^{Np}A_j^{Np} \\ -(\sigma_c^{Pu}+\sigma_f^{Pu})A_j^{Pu} \end{bmatrix} \bar{v}.$$

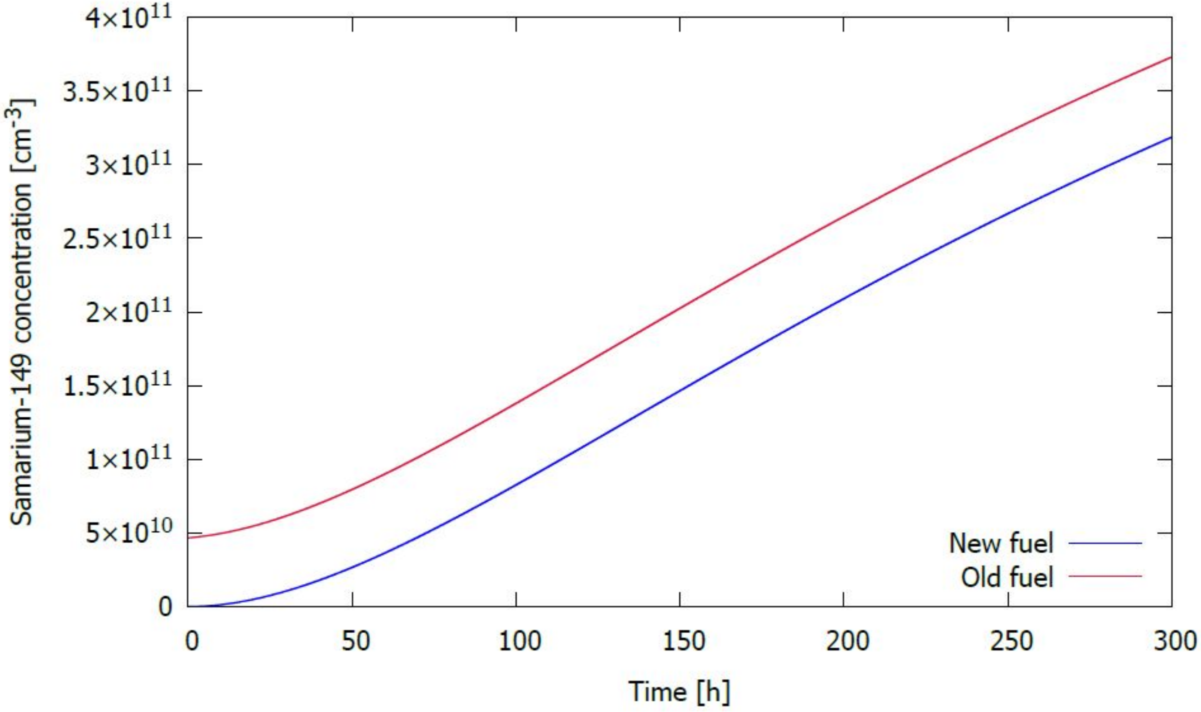
Adomian Polynomials

$$P_j = \frac{1}{j!}\left[\frac{d^j}{dw^j} N\left(\sum_{k=0}^{j} y_k^i w^k\right)\right]\Bigg|_{w=0}$$

$$B_j^i = G_i(t_0) \cdot \frac{1}{j!}\left[\frac{d^j}{dw^j} N\left(\sum_{k=0}^{j} n_k^i(t) w^k\right)\right]\Bigg|_{w=0}$$

$$A_j^{ab} = \frac{1}{j!}\left[\frac{d^j}{dw^j} N\left(\sum_{k=0}^{j} y_k^a w^k, \sum_{k=0}^{j} y_k^b w^k\right)\right]\Bigg|_{w=0}$$

Samarium-149 concentration



Uranium-235 concentration

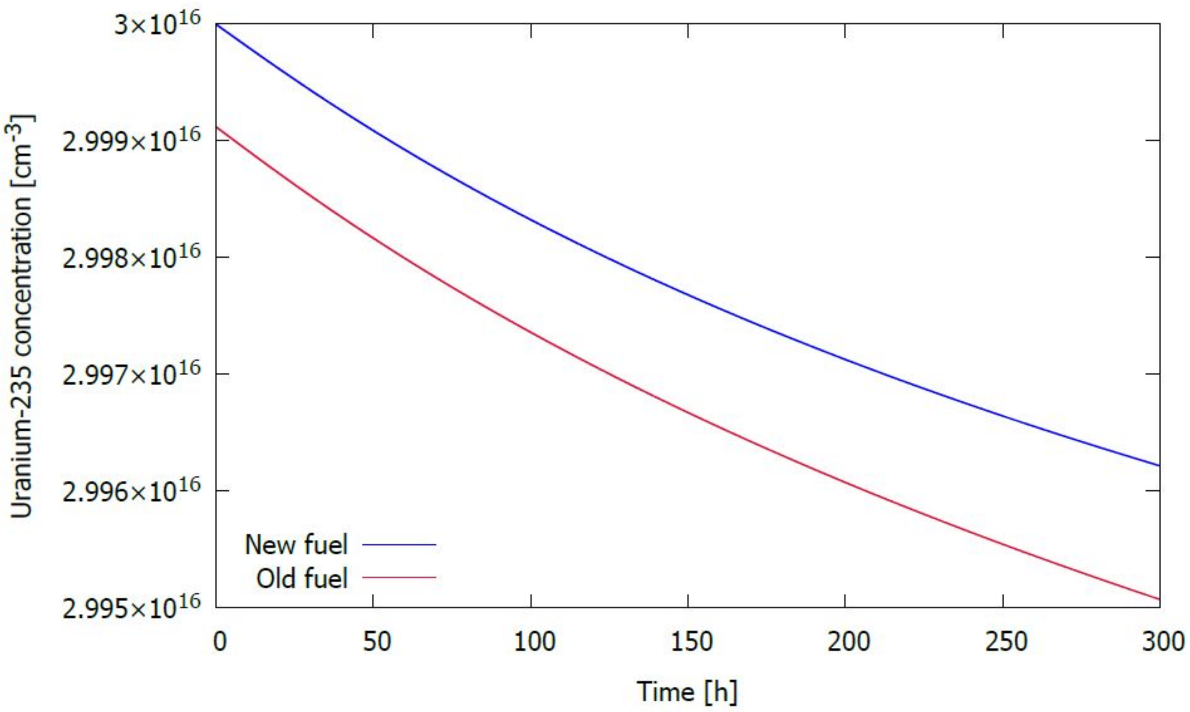


Table 1: Initial values.

	Case A	Case B
$n(t_0)$	$1 \cdot 10^9$	$1 \cdot 10^9$
$C(t_0)$	$8.397 \cdot 10^{11}$	$8.397 \cdot 10^{11}$
$G_I(t_0)$	0	$1.149 \cdot 10^{10}$
$G_{Xe}(t_0)$	0	$1.606 \cdot 10^{10}$
$G_{Pm}(t_0)$	0	$1.565 \cdot 10^{10}$
$G_{Sm}(t_0)$	0	$4.664 \cdot 10^{10}$
$G_{U_235}(t_0)$	$3 \cdot 10^{16}$	$2.9991 \cdot 10^{16}$
$G_{Np}(t_0)$	0	$2.314 \cdot 10^{11}$
$G_{Pu}(t_0)$	0	$6.414 \cdot 10^{11}$

Table 2: **Simulation paramaters.**

Λ	$2.7778 \cdot 10^{-8}$	$[h]$	γ_I^U	0.0629	σ_c^{Xe}	$2.6 \cdot 10^{-18}$	$[cm^2]$
λ	278.68	$[h^{-1}]$	γ_{Xe}^U	0.003	σ_c^{Sm}	$4.1 \cdot 10^{-20}$	$[cm^2]$
λ_I	0.1055	$[h^{-1}]$	γ_{Pm}^U	0.010816	σ_f^U	$582.6 \cdot 10^{-24}$	$[cm^2]$
λ_{Xe}	0.0762	$[h^{-1}]$	γ_I^{Pu}	0.06535	σ_f^{Pu}	$747.3 \cdot 10^{-24}$	$[cm^2]$
λ_{Pm}	0.013054	$[h^{-1}]$	γ_{Xe}^{Pu}	0.010763	σ_c^{U235}	$98.8 \cdot 10^{-24}$	$[cm^2]$
λ_{Np}	0.0123	$[h^{-1}]$	γ_{Pm}^{Pu}	0.012175	σ_c^{U238}	$2.719 \cdot 10^{-24}$	$[cm^2]$
λ_{Sm}^*	0.03	$[h^{-1}]$	ϵ	0.03	σ_c^{Np}	$68 \cdot 10^{-24}$	$[cm^2]$
λ_U^*	0.001	$[h^{-1}]$	ν_{Pu}	2.8836	$\bar{v}$	$1.1035 \cdot 10^7$	$[cm/h]$
λ_{Pu}^*	0.04	$[h^{-1}]$	ν_U	2.439	$\bar{\beta}$	0.0065	
λ_a^*	0.05	$[h^{-1}]$					

Convergence Analysis

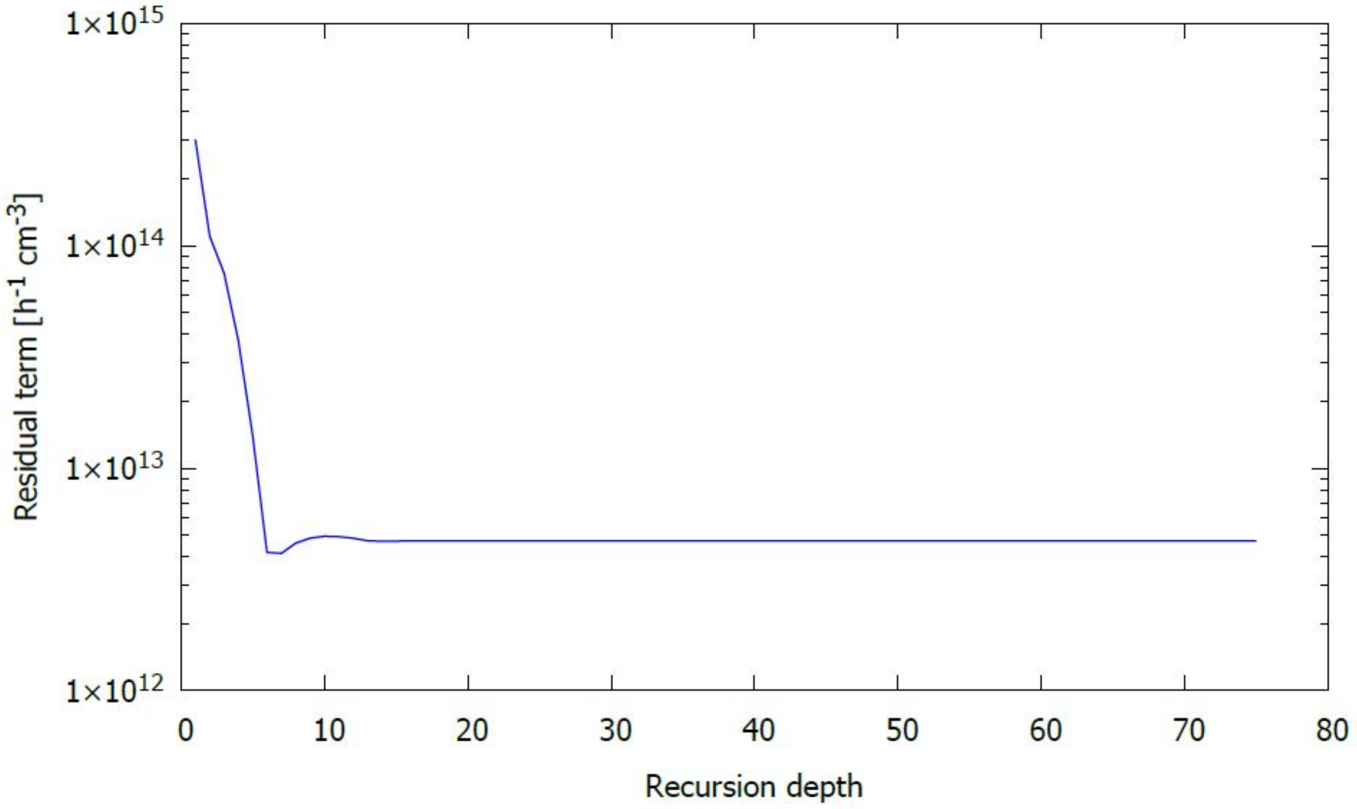
Table 3: Norm of the solution of the recursion j.

j	$\lvert n_j \rvert$	$\lvert C_j \rvert$	$\lvert G_j^{Xe} \rvert$	$\lvert G_j^{Sm} \rvert$	$\lvert G_j^{U} \rvert$	$\lvert G_j^{Pu} \rvert$
1	558180.1	$4.7 \cdot 10^8$	0.0	0.0	$3.0 \cdot 10^{16}$	0.0
2	$4.2 \cdot 10^6$	$6.1 \cdot 10^{10}$	$6.8 \cdot 10^9$	$2.1 \cdot 10^{10}$	$3.5 \cdot 10^{12}$	$2.1 \cdot 10^{11}$
3	$1.5 \cdot 10^7$	$1.3 \cdot 10^{10}$	$5.4 \cdot 10^8$	$5.3 \cdot 10^9$	$3.8 \cdot 10^{12}$	$5.1 \cdot 10^{10}$
10	$7.6 \cdot 10^7$	$6.4 \cdot 10^{10}$	$1.7 \cdot 10^{10}$	$1.1 \cdot 10^{10}$	$5.4 \cdot 10^{11}$	$2.0 \cdot 10^{11}$
50	147449.1	$9.4 \cdot 10^7$	$6.2 \cdot 10^7$	526928.0	$2.6 \cdot 10^8$	$4.6 \cdot 10^6$
100	120.066	41856.2	38744.9	62.8125	73722.3	310.991
200	$1.76 \cdot 10^{-5}$	0.0257	0.0192	$4.96 \cdot 10^{-5}$	0.0303	$2.77 \cdot 10^{-4}$

Table 4: Norm of the difference between consecutive solutions of the recursion j.

j	$\lvert n_j - n_{j-1} \rvert$	$\lvert C_j - C_{j-1} \rvert$	$\lvert G_j^{Xe} - G_{j-1}^{Xe} \rvert$
2	$3.6 \cdot 10^6$	$3.0 \cdot 10^9$	$1.2 \cdot 10^7$
3	$1.1 \cdot 10^7$	$9.6 \cdot 10^9$	$5.3 \cdot 10^8$
4	$2.3 \cdot 10^7$	$1.9 \cdot 10^{10}$	$1.9 \cdot 10^9$
10	$3.9 \cdot 10^7$	$3.3 \cdot 10^{10}$	$7.2 \cdot 10^9$
50	29304.2	$2.8 \cdot 10^7$	$1.3 \cdot 10^7$
100	48.2	28764.6	22030.8
200	$6.44 \cdot 10^{-5}$	0.01168	0.01195

j	$\lvert G_j^{Sm} - G_{j-1}^{Sm} \rvert$	$\lvert G_j^{U} - G_{j-1}^{U} \rvert$	$\lvert G_j^{Pu} - G_{j-1}^{Pu} \rvert$
2	0.0	$3.0 \cdot 10^{16}$	0.0
3	$5.3 \cdot 10^9$	$1.7 \cdot 10^{10}$	$7.9 \cdot 10^{10}$
4	$7.6 \cdot 10^9$	$6.8 \cdot 10^{10}$	$1.0 \cdot 10^{10}$
10	$5.5 \cdot 10^9$	$4.3 \cdot 10^{11}$	$6.0 \cdot 10^{10}$
50	264472.6	$1.1 \cdot 10^8$	$2.4 \cdot 10^6$
100	78.6	56107.7	528.4
200	$1.25 \cdot 10^{-5}$	0.020189	$3.43 \cdot 10^{-5}$



1×10^15
1×10^14
1×10^13
1×10^12
Residual term [h^-1 cm^-3]
0
10
20
30
40
50
60
70
80
Recursion depth