ON A NEUTRON POINT KINETIC MODEL CONSIDERING THE VARIATION IN THE FUEL COMPOSITION WITH CONVERGENCE ANALYSIS

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Summary

- Introduction
- Objectives
- Methodology
- Results
- Conclusion
Introduction

Classic Point kinetic model (short time scale).

Reactor fuel stays from 1 up to 3 years in the reactor core (long time scale).

Changes in the nuclear fuel material and properties over time.

Depletion codes - Quasi-static method: linearization of the problem, neutron flux approximation to constant in each time step.
Introduction

New model: Point kinetics and isotope depletion with reactivity decomposition in short and long time scales. Solved the nonlinear equation by the Adomian decomposition method. Considered only neutron poisons in the analysis.

Solves short, intermediate and long time scales together by the same method.

Does not need to approximate the neutron flux to a constant.
Objectives

Expand the model proposes to account Uranium-235 burn-up and transuranium (Plutonium-239 and Neptunium-239).

Convergence analysis of the solution.
Methodology

Classic point kinetic model:

\[
\frac{d}{dt} n(t) = \frac{\rho(t) - \bar{\beta}}{\Lambda} n(t) + \lambda C(t),
\]

\[
\frac{d}{dt} C(t) = \frac{\beta}{\Lambda} n(t) - \lambda C(t).
\]
Methodology

Isotope concentration equation:

\[ \frac{d}{dt} G_i(t) = \gamma_i \nu \sum_f n(t) + \lambda_j G_j(t) - \nu \sigma_{x,i} G_i(t) n(t) - \lambda_i G_i(t) \]

- increase concentration
- decrease concentration
Methodology

Decomposition of the reactivity:

\[ \rho(t) = \rho_s(t) + \rho_l(t) \]

- short time scale
- long time scale
Methodology

Decomposition of the reactivity:

\[ \rho(t) = \rho_s(t) + \rho_l(t) \]

short time scale

long time scale

Changes by the reactor operation maneuvers and control.

Variation of nuclear fuel composition.
Methodology

Simplifications:

- 1 energy group.
- 1 precursors group.
- Plutonium-239 delayed neutrons were not considered.
- Uranium-238 concentration and diffusion coefficient were considered constants.
- Fission cross-section is considered constant in the short time scale.

*These simplifications are not mandatory for the method. The model can be expanded.
Methodology

Adomian Decomposition Method:
- Analytical approximation to the solution by an infinite series.
- Solutions are obtained by solving recursively linear systems with source terms.
- The nonlinear term in the original equation is accounted as the source term.
- Nonlinear contributions are based on previous recursions solutions and calculated by Adomian Polynomials.
Methodology

The recursion initialization is a homogeneous linear system with the initial conditions.

\[
\begin{aligned}
\frac{d}{dt} Y_0 &= A Y_0 \\
Y_0(0) &= Y_I
\end{aligned}
\]

Analytical solution:

\[ Y_0(t) = e^{At} Y_I \]
Methodology

All other recursions:

\[
\begin{cases}
\frac{d}{dt} Y_j = A Y_j + F_{j-1} \\
Y_j(0) = 0
\end{cases}
\]

Analytical solution:

\[
Y_j(t) = e^{At} Y_j(0) + \int_0^t e^{A(t-\tau)} F_{j-1}(\tau) \, d\tau
\]
Results

New and old fuel as initial conditions.

Same neutron density and precursors concentration.

Short time scale reactivity is set critical. $\rho_s(t) = 0$
Neutron density

The graph shows the neutron density over time for two different fuel types: New fuel and Old fuel. The neutron density is measured in neutrons per cubic meter (cm$^{-3}$) and is plotted against time in hours (h). The graph indicates a decrease in neutron density over time, with the density starting from $1 \times 10^9$ neutrons/cm$^3$ at time 0 and decreasing to $4 \times 10^8$ neutrons/cm$^3$ at time 300 hours. The New fuel line is shown in blue, while the Old fuel line is shown in red. The New fuel line is slightly higher than the Old fuel line throughout the graph.
Xenon-135 concentration

![Xenon-135 concentration graph]

- **Y-axis**: Xenon-135 concentration [cm$^{-3}$]
- **X-axis**: Time [h]
- **Legend**:
  - New fuel (blue line)
  - Old fuel (red line)
Plutonium-239 concentration

![Graph showing Plutonium-239 concentration over time for New fuel and Old fuel.](image)
Convergence Analysis

The behavior of the solution indicates stability, however, it does not ensure it.

Residual term tends to a constant value orders of magnitude lower than the physical system.

Stiff system - Presents numerical and arithmetical instability. Parameters used were smaller than usual in point kinetic simulations.
Conclusion

This work contributes to expand this new approach to solve the classical problem of point kinetics and isotope composition variation in the nuclear fuel without using simplications in the reactivity and neutron flux, presenting a semi-analytical solution.

The Adomian decomposition method was used to solve the system of nonlinear differential equations. The results are consistent with those reported in the literature and what is expected from a physical point of view.
Conclusion

The solutions obtained through recursions present an apparently convergent behavior, however requires a more extensive analysis in order to better evaluate the convergence of the series.

The stiffness of the system causes computational instability, unabeling the simulation with higher initial condition parameters. Future works currently under development focus to solve this problem.
Conclusion

Future works:

Implement the code in a more suitable language, which has a better floating-point and arithmetic precision (C++).

More precise integration algorithms.

Multi-step Adomian Decomposition Method.
Thank you!

Other results presented in the Progress in Nuclear Energy:
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Other questions: tpaganin@gmail.com
APPENDIX
Methodology

\[
\rho(t) \equiv \frac{1}{F} \int_V \int_0^\infty \phi^*(r, E, t_0) \left( \int_0^\infty [\chi(E') \nu(E') \delta \Sigma_f(r, E', t) \\
+ \delta \Sigma_s(r, E' \rightarrow E, t)] f(r, E') dE' + \nabla \cdot [\delta D(r, E, t) \nabla f(r, E)] \\
- \delta \Sigma_t(r, E, t) f(r, E) \right) dEd^3r,
\]

\[
\delta \Sigma_x(r, E, t) \equiv \Sigma_x(r, E, t) - \Sigma_x(r, E, t_0) = \sum_{i=1}^I [G_i(r, t) - G_i(r, t_0)] \sigma_x^i(E)
\]
Methodology

\[ \rho(t) = \frac{1}{F} \int_V \int_0^\infty \phi^*(r, E, t_0) \left( \int_0^\infty [\chi(E')\nu(E')\delta \Sigma_f(r, E', t) \right. \]
\[ + \delta \Sigma_s(r, E' \rightarrow E, t)] f(r, E') dE' + \nabla \cdot [\delta D(r, E, t) \nabla f(r, E)] \]
\[ - \delta \Sigma_t(r, E, t) f(r, E) \right) dE d^3r, \]

\[ \delta \Sigma_x(r, E, t) \equiv \Sigma_x(r, E, t) - \Sigma_x(r, E, t_0) = \sum_{i=1}^I \left[ G_i(r, t) - G_i(r, t_0) \right] \sigma_x^i(E) \]
Source term

\[
F_j(t) = \begin{bmatrix}
    (\nu_{Pu} - 1)\sigma_f^{Pu} A_j^{Pu} + (\nu_u - 1)\sigma_f^{U235} A_j^{U235} - \sigma_c^{Xe} A_j^{Xe} - \sigma_c^{S_m} A_j^{S_m} \\
    -(\nu_u - 1)\sigma_f^{U235} B_j^{U235} + \sigma_c^{Xe} B_j^{Xe} + \sigma_c^{S_m} B_j^{S_m} - (\nu_{Pu} - 1)\sigma_f^{Pu} B_j^{Pu} \\
    0 \\
    \gamma_I^{U235} A_j^{Pu} + \gamma_I^{Pu} \sigma_f^{Pu} A_j^{Pu} \\
    \gamma_{Xe}^{U235} A_j^{Pu} + \gamma_{Xe}^{Pu} \sigma_f^{Pu} A_j^{Pu} - \sigma_c^{Xe} A_j^{Xe} \\
    \gamma_{Pm}^{U235} \sigma_f^{Pu} A_j^{Pu} + \gamma_{Pm}^{Pu} \sigma_f^{Pu} A_j^{Pu} - \sigma_c^{S_m} A_j^{S_m} \\
    -\sigma_f^{U235} A_j^{Pu} \\
    -\sigma_f^{Np} A_j^{Np} \\
    -\sigma_c^{Pm} A_j^{Pm} \\
    -(\sigma_c^{Pu} + \sigma_f^{Pu}) A_j^{Pu}
\end{bmatrix}
\]
Adomian Polynomials

\[
P_j = \frac{1}{j!} \left[ \frac{d^j}{dw^j} N \left( \sum_{k=0}^{j} y_k w^k \right) \right]_{w=0}
\]

\[
B_j^i = G_i(t_0) \cdot \frac{1}{j!} \left[ \frac{d^j}{dw^j} N \left( \sum_{k=0}^{j} n_k^i(t) w^k \right) \right]_{w=0}
\]

\[
A_{j}^{ab} = \frac{1}{j!} \left[ \frac{d^j}{dw^j} N \left( \sum_{k=0}^{j} y_k^a w^k, \sum_{k=0}^{j} y_k^b w^k \right) \right]_{w=0}
\]
Samarium-149 concentration

![Graph showing concentration of Samarium-149 over time for new and old fuel.](image)
Uranium-235 concentration
Table 1: Initial values.

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n(t_0) )</td>
<td>( 1 \cdot 10^9 )</td>
<td>( 1 \cdot 10^9 )</td>
</tr>
<tr>
<td>( C(t_0) )</td>
<td>( 8.397 \cdot 10^{11} )</td>
<td>( 8.397 \cdot 10^{11} )</td>
</tr>
<tr>
<td>( G_I(t_0) )</td>
<td>0</td>
<td>1.149 ( \cdot 10^{10} )</td>
</tr>
<tr>
<td>( G_{Xe}(t_0) )</td>
<td>0</td>
<td>1.606 ( \cdot 10^{10} )</td>
</tr>
<tr>
<td>( G_{Pm}(t_0) )</td>
<td>0</td>
<td>1.565 ( \cdot 10^{10} )</td>
</tr>
<tr>
<td>( G_{Sm}(t_0) )</td>
<td>0</td>
<td>4.664 ( \cdot 10^{10} )</td>
</tr>
<tr>
<td>( G_{U235}(t_0) )</td>
<td>( 3 \cdot 10^{16} )</td>
<td>2.9991 ( \cdot 10^{16} )</td>
</tr>
<tr>
<td>( G_{NP}(t_0) )</td>
<td>0</td>
<td>2.314 ( \cdot 10^{11} )</td>
</tr>
<tr>
<td>( G_{Pu}(t_0) )</td>
<td>0</td>
<td>6.414 ( \cdot 10^{11} )</td>
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Table 2: Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>(\Lambda)</td>
<td>2.7778 \cdot 10^{-8}</td>
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<tr>
<td>(\lambda)</td>
<td>278.68</td>
<td>[h(^{-1})]</td>
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<tr>
<td>(\lambda_I)</td>
<td>0.1055</td>
<td>[h(^{-1})]</td>
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<tr>
<td>(\lambda_{Xe})</td>
<td>0.0762</td>
<td>[h(^{-1})]</td>
</tr>
<tr>
<td>(\lambda_{Pm})</td>
<td>0.013054</td>
<td>[h(^{-1})]</td>
</tr>
<tr>
<td>(\lambda_{NP})</td>
<td>0.0123</td>
<td>[h(^{-1})]</td>
</tr>
<tr>
<td>(\lambda^*_{Sm})</td>
<td>0.03</td>
<td>[h(^{-1})]</td>
</tr>
<tr>
<td>(\lambda^*_U)</td>
<td>0.001</td>
<td>[h(^{-1})]</td>
</tr>
<tr>
<td>(\lambda^*_{Pu})</td>
<td>0.04</td>
<td>[h(^{-1})]</td>
</tr>
<tr>
<td>(\lambda^*_n)</td>
<td>0.05</td>
<td>[h(^{-1})]</td>
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<tr>
<td>(\gamma^U)</td>
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<tr>
<td>(\gamma^U_{I})</td>
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<tr>
<td>(\gamma^U_{Xe})</td>
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<td>(\gamma^U_{Pm})</td>
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<td>(\gamma^P)</td>
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<td>(\gamma^P_{Pu})</td>
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<td>(\gamma^P_{Pu})</td>
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<td>(\nu^P)</td>
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<td>(\nu_{Pu})</td>
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<tr>
<td>(\nu_U)</td>
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<td>(\sigma^{Xe}_c)</td>
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<td>[cm(^2)]</td>
</tr>
<tr>
<td>(\sigma^{Sm}_c)</td>
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<td>[cm(^2)]</td>
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<td>(\sigma_c^U)</td>
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<td>[cm(^2)]</td>
</tr>
<tr>
<td>(\sigma_c^{Pu})</td>
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<td>[cm(^2)]</td>
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<td>(\sigma_{cU_{235}})</td>
<td>98.8 \cdot 10^{-24}</td>
<td>[cm(^2)]</td>
</tr>
<tr>
<td>(\sigma_{cU_{238}})</td>
<td>2.719 \cdot 10^{-24}</td>
<td>[cm(^2)]</td>
</tr>
<tr>
<td>(\sigma_{cNP})</td>
<td>68 \cdot 10^{-24}</td>
<td>[cm(^2)]</td>
</tr>
<tr>
<td>(\bar{\nu})</td>
<td>1.1035 \cdot 10^7</td>
<td>[cm/h]</td>
</tr>
<tr>
<td>(\bar{\beta})</td>
<td>0.0065</td>
<td></td>
</tr>
</tbody>
</table>
Convergence Analysis

Table 3: Norm of the solution of the recursion $j$.

| $j$ | $|n_j|$ | $|C_j|$ | $|G_{j}^{Xe}|$ | $|G_{j}^{Sm}|$ | $|G_{j}^{U}|$ | $|G_{j}^{Pu}|$ |
|-----|--------|--------|----------------|----------------|-------------|-------------|
| 1   | 558180.1 | 4.7 $\cdot 10^8$ | 0.0 | 0.0 | 3.0 $\cdot 10^{16}$ | 0.0 |
| 2   | $4.2 \cdot 10^6$ | 6.1 $\cdot 10^{10}$ | 6.8 $\cdot 10^9$ | 2.1 $\cdot 10^{10}$ | 3.5 $\cdot 10^{12}$ | 2.1 $\cdot 10^{11}$ |
| 3   | $1.5 \cdot 10^7$ | 1.3 $\cdot 10^{10}$ | 5.4 $\cdot 10^8$ | 5.3 $\cdot 10^9$ | 3.8 $\cdot 10^{12}$ | 5.1 $\cdot 10^{10}$ |
| 10  | $7.6 \cdot 10^7$ | 6.4 $\cdot 10^{10}$ | 1.7 $\cdot 10^{10}$ | 1.1 $\cdot 10^{10}$ | 5.4 $\cdot 10^{11}$ | 2.0 $\cdot 10^{11}$ |
| 50  | 147449.1 | 9.4 $\cdot 10^7$ | 6.2 $\cdot 10^7$ | 526928.0 | 2.6 $\cdot 10^8$ | 4.6 $\cdot 10^6$ |
| 100 | 120.066 | 41856.2 | 38744.9 | 62.8125 | 73722.3 | 310.991 |
| 200 | $1.76 \cdot 10^{-5}$ | 0.0257 | 0.0192 | 4.96 $\cdot 10^{-5}$ | 0.0303 | 2.77 $\cdot 10^{-4}$ |
Table 4: Norm of the difference between consecutive solutions of the recursion \( j \).

| \( j \) | \( |n_j - n_{j-1}| \) | \( |C_j - C_{j-1}| \) | \( |G_j^{Xe} - G_{j-1}^{Xe}| \) |
|---|---|---|---|
| 2 | 3.6 \cdot 10^6 | 3.0 \cdot 10^9 | 1.2 \cdot 10^7 |
| 3 | 1.1 \cdot 10^7 | 9.6 \cdot 10^9 | 5.3 \cdot 10^8 |
| 4 | 2.3 \cdot 10^7 | 1.9 \cdot 10^{10} | 1.9 \cdot 10^9 |
| 10 | 3.9 \cdot 10^7 | 3.3 \cdot 10^{10} | 7.2 \cdot 10^9 |
| 50 | 29304.2 | 2.8 \cdot 10^7 | 1.3 \cdot 10^7 |
| 100 | 48.2 | 28764.6 | 22030.8 |
| 200 | 6.44 \cdot 10^{-5} | 0.01168 | 0.01195 |

| \( j \) | \( |G_j^{Sm} - G_{j-1}^{Sm}| \) | \( |G_j^{U} - G_{j-1}^{U}| \) | \( |G_j^{Pu} - G_{j-1}^{Pu}| \) |
|---|---|---|---|
| 2 | 0.0 | 3.0 \cdot 10^{16} | 0.0 |
| 3 | 5.3 \cdot 10^9 | 1.7 \cdot 10^{10} | 7.9 \cdot 10^{10} |
| 4 | 7.6 \cdot 10^9 | 6.8 \cdot 10^{10} | 1.0 \cdot 10^{10} |
| 10 | 5.5 \cdot 10^9 | 4.3 \cdot 10^{11} | 6.0 \cdot 10^{10} |
| 50 | 264472.6 | 1.1 \cdot 10^8 | 2.4 \cdot 10^6 |
| 100 | 78.6 | 56107.7 | 528.4 |
| 200 | 1.25 \cdot 10^{-5} | 0.020189 | 3.43 \cdot 10^{-5} |