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MULTIPLICATION FACTOR (k_{eff}) SENSITIVITY ANALYSIS IN RELATION TO THE VOID OF A FAST BREEDER REACTOR

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Summary

1. Introduction
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1. Introduction

- Nuclear reactor design requires complex numerical calculations due to the detailed geometric structure and composition variation.

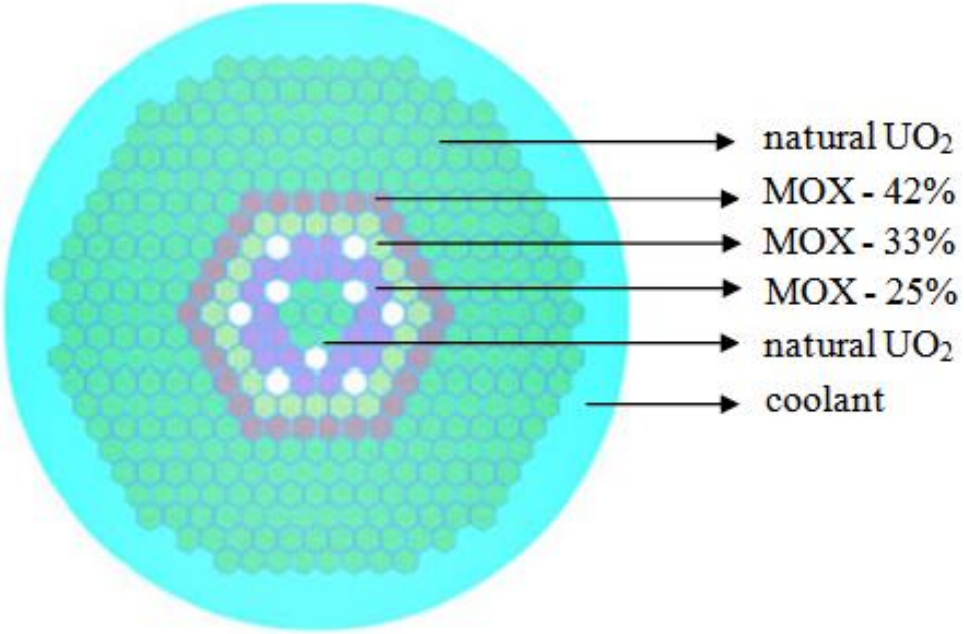
- **Analytical solution permits a better understanding of the response to a specific parameter perturbation.**

- A nuclear core design starts with a simple model so the equations can be solved without numerical methods.

1. Introduction

- This work focus on an analytical model considering a sphere geometry, compare the results of SCALE/KENO VI code with those from diffusion approximation for one energy group obtained with a FORTRAN program.

2. Fast Breeder Reactor (FBR)



- Rg A and E - natural uranium.

Oxide Moisture - MOX:

- Rg B - PuO_2 - 25% - UO_2.- 75%;
- Rg C - PuO_2 - 33% - UO_2.- 67%;
- Rg D - PuO_2 - 42% - UO_2.- 58%.

- The FBR is cooled by sodium liquid, has a hexagonal prism form, a heterogeneous core with five regions, height and diameter of 180cm.
- Fuel PuO_2 its main characteristic, independent of enriched uranium.

3. Methodology

- One energy group diffusion analytical study;
- One dimensional system;
- Five concentric spheres.

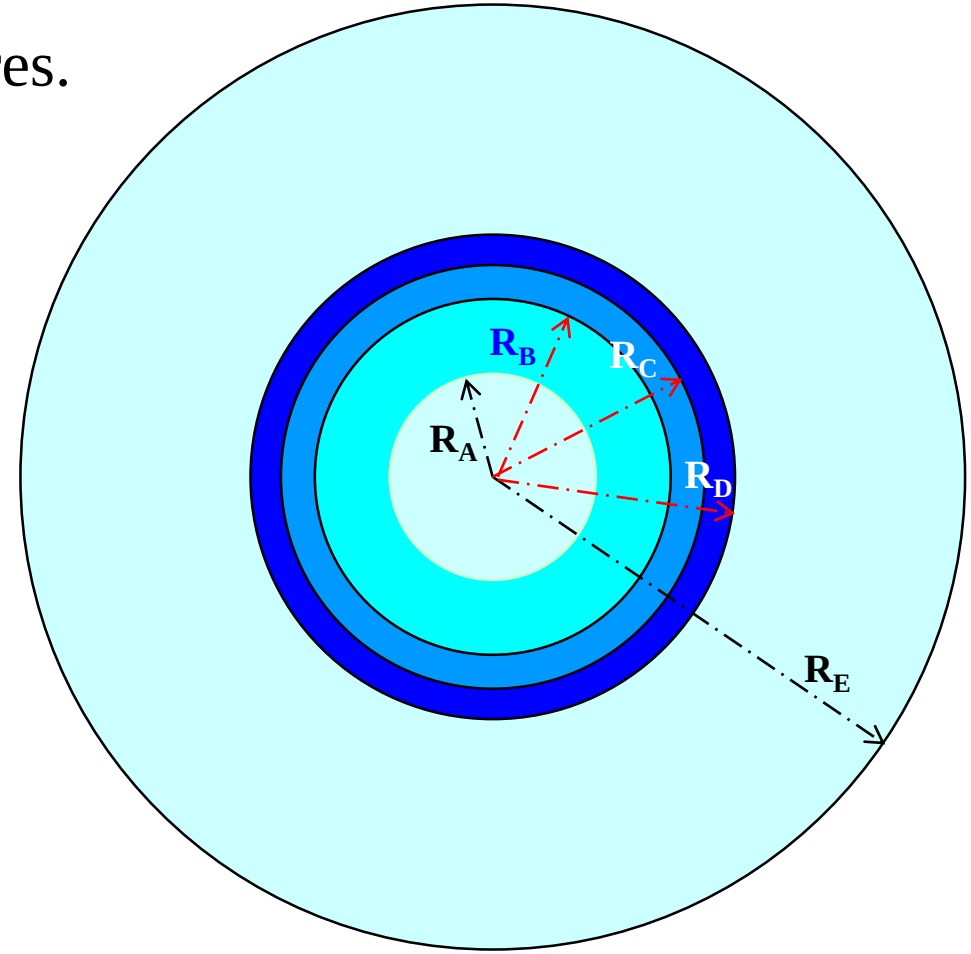
R_A = 20.582cm;

R_B = 35.583cm;

R_C = 42.354cm;

R_D = 48.395cm;

R_E = 94.429cm.



Spheric coordenates:

$$\nabla^2 \Phi = \frac{d^2 (r\Phi)}{r dr^2}$$

3. Methodology

Analitical study: one energy group.

Reactor governing equations:

$$-D_i \nabla^2 \Phi_i + \Sigma a_i \Phi_i = \frac{\nu_i \Sigma_{f_i}}{k_{eff}} \Phi_i; \; i = A, \; B, \; C, \; D \; and \; E$$

$$\nabla^2 \Phi_i + \frac{1}{D_i} \left[\frac{\nu_i \Sigma_{f_i}}{k_{eff}} - \Sigma_{a_i} \right] \Phi_i = 0;$$

D_i - diffusion coefficient;
∇^2 - laplacian operator;
Φ_i - flux distribution;
Σ_{a_i} - absorption macroscopic cross section;
Σ_{f_i}- fission macroscopic cross section;
ν_i - mean neutrons released in each fission;
k_{eff} - effective multiplication factor.

3. Methodology

Analitical study: one energy group.

Reactor governing equations:

$$\nabla^2 \Phi_i + {K_i}^2 \Phi_i = 0$$

where ${K_i}^2 = \frac{1}{D_i}\left[\frac{\nu_i \Sigma_{f_i}}{k_{eff}} - \Sigma_{a_i}\right]$.

For regions A and E, $K_i^2 < 0$ and regions B, C and D, $K_i^2 > 0$.

With only radial dependence governing equation can be written for each region as:

$$\frac{1}{r}\frac{d^2(r\Phi_i)}{dr^2} + {K_i}^2 \Phi_i = 0$$

3. Methodology

Analitical study: one energy group.

The solution for each region is:

$$\Phi_A = C_1 \frac{\sinh(K_1 r)}{r} + C_2 \frac{\cosh(K_1 r)}{r}$$

$$\Phi_B = C_3 \frac{\sin(K_2 r)}{r} + C_4 \frac{\cos(K_2 r)}{r}$$

$$\Phi_C = C_5 \frac{\sin(K_3 r)}{r} + C_6 \frac{\cos(K_3 r)}{r}$$

$$\Phi_D = C_7 \frac{\sin(K_4 r)}{r} + C_8 \frac{\cos(K_4 r)}{r}$$

$$\Phi_E = C_9 \frac{\sinh(K_5 r)}{r} + C_{10} \frac{\cosh(K_5 r)}{r}$$

3. Methodology

Analitical study - boundary conditions:

Considering the neutron flux Φ_A finite at $r = 0$ (i), so we have $C_2 = 0$; thus:

$$\Phi_A = C_1 \frac{\sinh(K_1 r)}{r}$$

Also, considering the neutron flux and current density equality in the regions frontiers, follows:

Flux continuity	*Current continuity*
$(ii)\ \Phi_A\ (R_A) = \Phi_B\ (R_A)$	$(iii)\ D_A\ \frac{d\Phi_A}{dr}\ (R_A) = D_B\ \frac{d\Phi_B}{dr}\ (R_A)$
$(iv)\ \Phi_B\ (R_B) = \Phi_C\ (R_B)$	$(v)\ D_B\ \frac{d\Phi_B}{dr}\ (R_B) = D_C\ \frac{d\Phi_C}{dr}\ (R_B)$
$(vi)\ \Phi_C\ (R_C) = \Phi_D\ (R_C)$	$(vii)\ D_C\ \frac{d\Phi_C}{dr}\ (R_c) = D_D\ \frac{d\Phi_D}{dr}\ (R_C)$
$(viii)\ \Phi_D\ (R_D) = \Phi_E\ (R_D)$	$(ix)\ D_D\ \frac{d\Phi_D}{dr}\ (R_D) = D_E\ \frac{d\Phi_E}{dr}\ (R_D)$

3. Methodology

Analitical study - boundary conditions:

For $r = R_E$, the partial neutron reentering current is zero and we may write it as:

$$(x)\ J_{-E} = 0 = \frac{\Phi_E}{4} + \frac{D_E}{2}\frac{d\Phi_E}{dr}$$

The normalization condition is defined as bellow (xi):

$$\int_{r=0}^{r_A}(-D_A\nabla^2\Phi_A + \Sigma a_A\Phi_A)dv \ + \int_{r=r_A}^{r_B}(-D_B\nabla^2\Phi_B + \Sigma a_B\Phi_B)dv \ +$$
$$\int_{r=r_B}^{r_C}(-D_C\nabla^2\Phi_C + \Sigma a_C\Phi_C)dv \ + \int_{r=r_C}^{r_D}(-D_D\nabla^2\Phi_D + \Sigma a_D\Phi_D)dv \ +$$
$$\int_{r=r_D}^{r_E}(-D_E\nabla^2\Phi_E + \Sigma a_E\Phi_E) \ = \frac{1}{s}$$

where, $dv = 4\pi r^2 dr$.

3. Methodology

Analitical study - k_{eff} calculation:

To calculate k_{eff} in this new approach we take diffusion equation.

Integrating over the whole reactor volume we get:

$$k_{eff} = A_A \frac{\nu_A \Sigma_{f_A}}{\Sigma_{a_A}} + A_B \frac{\nu_B \Sigma_{f_B}}{\Sigma_{a_B}} + A_C \frac{\nu_C \Sigma_{f_C}}{\Sigma_{a_C}} + A_D \frac{\nu_D \Sigma_{f_D}}{\Sigma_{a_D}} + A_E \frac{\nu_E \Sigma_{f_E}}{\Sigma_{a_E}}$$

where,

$$A_A = \int_{r=0}^{r_A} \Sigma_{a_A} \Phi_A dv;\ A_B = \int_{r=r_A}^{r_B} \Sigma_{a_B} \Phi_B dv;\ A_C = \int_{r=r_B}^{r_C} \Sigma_{a_C} \Phi_C dv;$$

$$A_D = \int_{r=r_C}^{r_D} \Sigma_{a_D} \Phi_D dv;\ A_E = \int_{r=r_D}^{r_E} \Sigma_{a_E} \Phi_E dv.$$

3. Methodology

Analitical study - reactivity coefficient calculation:

$$\alpha = \frac{\rho_{Vd} - \rho_{NVd}}{\frac{V_{Vd}}{V_{cool}}}$$

where,

$\rho_{Vd} = \frac{k_{eff_{Vd}}{}^{-1}}{k_{eff_{Vd}}}$ reactivity with void;

$\rho_{NVd} = \frac{k_{eff_{NVd}}{}^{-1}}{k_{eff_{NVd}}}$ reactivity without void;

V_{Vd} void volume;

V_{cool} coolant volume.

3. Methodology

- A FORTRAN language program was built to perform calculations in order to obtain results concerning to neutron absorption, leakage, flux radial distribution, the reactor k_{eff} and reactivity coefficient values.

- The results were analyzed and compared with those obtained using SCALE code.

3. Results

Group constants calculated by the SCALE code.

Tables 1 - Group constants considering no void.

	Region A	Region B	Region C	Region D	Region E
$^{(a)}\Sigma_{tr}$	0.21325	0.19206	0.18725	0.18241	0.21325
$^{(b)}\Sigma_{a}$	3.93154E-03	6.91455E-03	7.89135E-03	9.02257E-03	3.93154E-03
$^{(c)}\nu\Sigma_{f}$	1.37568E-03	1.23526E-02	1.57379E-02	1.96054E-02	1.37568E-03

Tables 2 - Group constants considering 5.87% of void on each region.

	Region A	Region B	Region C	Region D
$^{(a)}\Sigma_{tr}$	0.15899	0.17853	0.17320	0.17138
$^{(b)}\Sigma_{a}$	3.36409E-03	6.77542E-03	7.75785E-03	1.96088E-02
$^{(c)}\nu\Sigma_{f}$	1.29415E-03	1.23005E-02	1.57094E-02	1.96088E-02

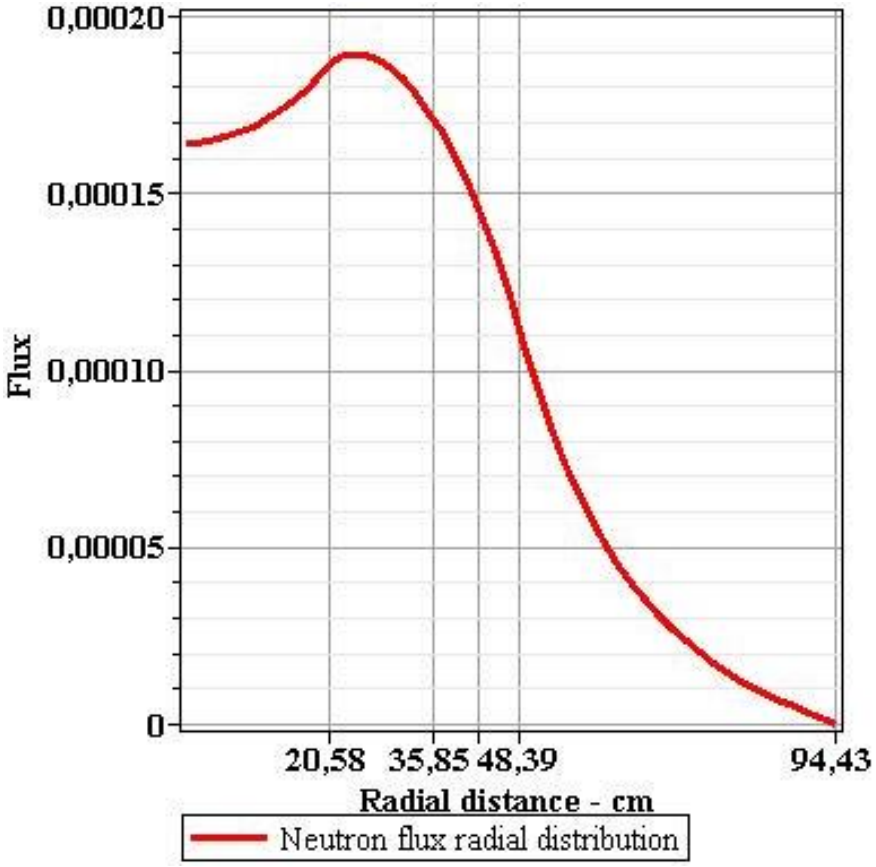
$^{(a)}\sum_{tr}$ - transport cross section;

$^{(b)}\sum_{a}$ - absorption cross section;

$^{(c)}\nu\sum_{f}$ - mean neutrons released in each fission and fission cross section.

3. Results

Neutron flux radial distribution for the reactor core without void inserted. The maximum neutron flux value is observed on region B and decreases until near zero at radius equal R_E.



3. Results

	No Void	Void A	Void B	Void C	Void D
[a] A_A	2.54899E-02	2.25959E-02	2.55832E-02	2.53465E-02	2.53885E-02
A_B	0.19398	0.19559	0.19070	0.19292	0.19311
A_C	0.16453	0.16504	0.16539	0.16178	0.16362
A_D	0.18547	0.18580	0.18625	0.18668	0.18359
A_E	**0.31548**	**0.31584**	**0.31665**	**0.31751**	**0.31823**
[b] A_{To}	**0.88495**	**0.88488**	**0.88458**	**0.88424**	**0.88395**
[c] F	**0.11505**	**0.11512**	**0.11542**	**0.11576**	**0.11605**
[d] k_{effA}	8.91915E-03	8.69252E-03	8.95177E-03	8.86896E-03	8.88366E-03
k_{effB}	0.34653	0.34943	0.34621	0.34465	0.34499
k_{effC}	0.32813	0.32915	0.32984	0.32761	0.32631
k_{effD}	**0.40302**	**0.40373**	**0.40471**	**0.40564**	**0.40322**
k_{effE}	0.11039	0.11052	0.11079	0.11109	0.11135
[e] k_{effCa}	**1.19698**	**1.20153**	**1.20051**	**1.19786**	**1.19476**
[f] k_{effIn}	1.19698	1.20152	1.20051	1.19786	1.19476
[g] k_{effSc}	1.1465	1.1467	1.1457	1.1455	1.1448
[h] δ_{kSc-Ca}	**4.40**	**4.78**	**4.78**	**4.57**	**4.36**
[i] α_{Ca}	-x-x-x-	0.05379	0.04186	0.01042	-0.02653
[j] α_{Sc}	-x-x-x-	0.00302	-0.00951	-0.01211	-0.02163

[a] - Absorptions on regions A, B, C, D and E
[b] - Total absorption
[c] - Neutron leakage
[d] - k_{eff} partial for regions A, B, C, D and E
[e] - k_{eff} calculated by FORTRAN program
[f] - k_{eff} inserted in the FORTRAN program
[g] - k_{eff} calculated by SCALE code
[h] - δ_{kS} (%) deviation between k_{eff}Sc and k_{eff}Ca
[i] - Alpha-α calculated by FORTRAN program
[j] - Alpha-α calculated by SCALE code

3. Results

	No Void	Void A	Void B	Void C	Void D
[a] A_A	2.54899E-02	2.25959E-02	2.55832E-02	2.53465E-02	2.53885E-02
A_B	0.19398	0.19559	0.19070	0.19292	0.19311
A_C	0.16453	0.16504	0.16539	0.16178	0.16362
A_D	0.18547	0.18580	0.18625	0.18668	0.18359
A_E	**0.31548**	**0.31584**	**0.31665**	**0.31751**	**0.31823**
[b] A_{To}	**0.88495**	**0.88488**	**0.88458**	**0.88424**	**0.88395**

- Total absorption maximum value occurs for the condition without void (0.88495).
- As the void is inserted the respective absorption value decreases assuming the minimum value on region D.

[a] - Absorptions on regions A, B, C, D and E
[b] - Total absorption
[c] - Neutron leakage
[d] - k_{eff} partial for regions A, B, C, D and E
[e] - k_{eff} calculated by FORTRAN program
[f] - k_{eff} inserted in the FORTRAN program
[g] - k_{eff} calculated by SCALE code
[h] - δ_{kS} (%) deviation between k_{eff}Sc and k_{eff}Ca
[i] - Alpha-α calculated by FORTRAN program
[j] - Alpha-α calculated by SCALE code

3. Results

	No Void	Void A	Void B	Void C	Void D
[b] A_{To}	0.88495	0.88488	0.88458	0.88424	0.88395
[c] F	0.11505	0.11512	0.11542	0.11576	0.11605

- The neutron leakage increases from the value of 0.11505 on the condition without void until the maximum one 0.11605 for the condition with void on region D.

[a] - Absorptions on regions A, B, C, D and E
[b] - Total absorption
[c] - Neutron leakage
[d] - k_{eff} partial for regions A, B, C, D and E
[e] - k_{eff} calculated by FORTRAN program
[f] - k_{eff} inserted in the FORTRAN program
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[h] - δ_{kS} (%) deviation between k_{eff}Sc and k_{eff}Ca
[i] - Alpha-α calculated by FORTRAN program
[j] - Alpha-α calculated by SCALE code

3. Results

	No Void	Void A	Void B	Void C	Void D

- The maximum k_{eff} value calculated (1.20153).

- As void is inserted on the other regions (B, C and D) the k_{eff} value calculated decreases.

[d] k_{effA}	8.91915E-03	8.69252E-03	8.95177E-03	8.86896E-03	8.88366E-03
k_{effB}	0.34653	0.34943	0.34621	0.34465	0.34499
k_{effC}	0.32813	0.32915	0.32984	0.32761	0.32631
k_{effD}	**0.40302**	**0.40373**	**0.40471**	**0.40564**	**0.40322**
k_{effE}	0.11039	0.11052	0.11079	0.11109	0.11135
[e] k_{effCa}	**1.19698**	**1.20153**	**1.20051**	**1.19786**	**1.19476**

- Is important observe that all of them are bigger than the k_{eff} value calculated for the condition without void (1.19698), excepted k_{eff} value calculated for void inserted on region D (1.19476).

[a] - Absorptions on regions A, B, C, D and E
[b] - Total absorption
[c] - Neutron leakage
[d] - k_{eff} partial for regions A, B, C, D and E
[e] - k_{eff} calculated by FORTRAN program
[f] - k_{eff} inserted in the FORTRAN program
[g] - k_{eff} calculated by SCALE code
[h] - δ_{kS} (%) deviation between k_{eff}Sc and k_{eff}Ca
[i] - Alpha-α calculated by FORTRAN program
[j] - Alpha-α calculated by SCALE code

3. Results

	No Void	Void A	Void B	Void C	Void D

- The k_{eff} value by SCALE code has the same behavior.

$^{(e)}k_{effCa}$	**1.19698**	**1.20153**	**1.20051**	**1.19786**	**1.19476**
$^{(g)}k_{effSc}$	1.1465	1.1467	1.1457	1.1455	1.1448

(a) - Absorptions on regions A, B, C, D and E
(b) - Total absorption
(c) - Neutron leakage
(d) - k_{eff} partial for regions A, B, C, D and E
(e) - k_{eff} calculated by FORTRAN program
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(g) - k_{eff} calculated by SCALE code
(h) - δ_{kS} (%) deviation between k_{eff}Sc and k_{eff}Ca
(i) - Alpha-α calculated by FORTRAN program
(j) - Alpha-α calculated by SCALE code

3. Results

	No Void	Void A	Void B	Void C	Void D

- Comparing the k_{eff} values calculated by the FORTRAN program and those by SCALE code the results show a deviation (δ_{ks}) that varies from 4.36% (with void on region D) to 4.78% (with void on region B).

$^{(e)}k_{effCa}$	**1.19698**	**1.20153**	**1.20051**	**1.19786**	**1.19476**
$^{(g)}k_{effSc}$	1.1465	1.1467	1.1457	1.1455	1.1448
$^{(h)}\delta_{kSc-Ca}$	**4.40**	**4.78**	**4.78**	**4.57**	**4.36**

(a) - Absorptions on regions A, B, C, D and E
(b) - Total absorption
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(d) - k_{eff} partial for regions A, B, C, D and E
(e) - k_{eff} calculated by FORTRAN program
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(g) - k_{eff} calculated by SCALE code
(h) - δ_{kS} (%) deviation between k_{eff}Sc and k_{eff}Ca
(i) - Alpha-α calculated by FORTRAN program
(j) - Alpha-α calculated by SCALE code

3. Results

	No Void	Void A	Void B	Void C	Void D

- The void reactivity coefficient values calculated by the FORTRAN program and by SCALE code show the same behavior, decreasing values as void moves away from the nuclear reactor center.

$^{(i)}\alpha_{Ca}$	-x-x-x-	0.05379	0.04186	0.01042	-0.02653
$^{(j)}\alpha_{Sc}$	-x-x-x-	0.00302	-0.00951	-0.01211	-0.02163

(a) - Absorptions on regions A, B, C, D and E
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(h) - δ_{kS} (%) deviation between k_{eff}Sc and k_{eff}Ca
(i) - Alpha-α calculated by FORTRAN program
(j) - Alpha-α calculated by SCALE code

4. Conclusions

- The analytical method is simple in computation and permit to be made rapidly.

- Void insertion hardens the spectrum increasing the number of neutrons produced by fission per neutron absorbed (η factor). If the leakage does not compensate this effect we will have a positive void reactivity coefficient.

- This is well observed at reactor central region and the reactivity coefficient becomes positive.

- For region D, the leakage is dominant and the reactivity coefficient turns to negative.

4. Conclusions

- The model can predict the void reactivity coefficient tendency (positive or negative) and the multiplication factor deviation is less than 5% from real reactor core.

- Therefore, we conclude that this approach is a powerful tool for the core reactor design initial steps.

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Thank you!!!

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